PACKING CALCULATIONS APPLIED FOR CONCRETE MIX DESIGN

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ABSTRACT. When selecting a concrete mix design, it is always desirable to compose the aggregates as densely as possible, i.e. with maximum packing. That minimises the necessary amount of binder which has to fill the cavities between the aggregates for a constant concrete workability. Apart from an obvious economic benefit, a minimum of binder in concrete results in less shrinkage and creep and a more dense and therefore probably a more durable and strong concrete type. The paper presents a packing model which can be used to optimise the aggregate, both the theory behind, the available computer program and the necessary experimental measurements. Further, the application of packing calculations for concrete mix design to select the types and amounts of materials, to select the amount of binder, and to design the air void system will be described

Keywords: Packing, Binder, Mix design, Computer program, Experimental packing, Air void system.

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PACKING OF CONCRETE MATERIALS

When selecting a concrete mix design, it is always desirable to compose the aggregates as densely as possible, i.e. with maximum packing. That minimises the necessary amount of binder which has to fill the cavities between the aggregates for a constant concrete workability, see for instance [1]. Apart from an obvious economic benefit, a minimum of binder in concrete results in less shrinkage and creep and a more dense and therefore probably a more durable and strong concrete type.

Another important result of a good packing of the aggregate and the consequent less amount of binder is the economic saving due to lower cement content.

Theoretically, there is an unlimited number of possibilities for composing the aggregates, and in practice it is impossible to evaluate the effect of all these possibilities. This is the background for seeking help in packing models which can calculate the packing of any combination of materials.

The packing approach for concrete mix design differs from the traditional concrete mix design approach with simple and empirical hand rules and design principles, mainly with the basis in practical experience. Of these can be mentioned: water need as a function of slump, sand and stone composition from reference particle size distributions and a sand % at 40. The latter is confirmed by packing calculations.

Definition of packing

Packing can be defined as the volume of particles in relation to the total volume or as one minus the porosity.

PACKING MODEL

Historical review

The concept of particle packing is not new. Already in 1907, Fuller and Thompson, [2], investigated the importance of the size distribution of the aggregates and the properties of the concrete on the basis of packing of constituent materials.

Suenson, [3], presented in 1911 experimentally based diagrams of packing of the aggregates. These diagrams look like the triangular packing diagrams which are the results of the computer-based packing program described in this article, see Figure 1. Powers, [4], must also be mentioned for his extensive work with regard to concrete mix design on the basis of packing. More recently, Bache, [5], has been arguing for the use of the concept of packing for concrete mix design.

Basic research of packing theory was started by Furnas in 1931, [6]. His theory was set up for sphere shaped particles and was based on the assumption that the small particles fill out the cavities between the big particles without disturbing the packing of the big particles.

Since Furnas, the packing models have been developed. The models have developed from only considering sphere shaped particles, systems with two or three particle sizes without interaction to include systems with many particle sizes or with a continuous distribution including interaction phenomenon. It is especially the development within the computer technology that has promoted the development of packing models. A detailed description of packing models can be seen in Larsén, [7].

Apart from the packing model, which is the basis for this article, another computerised packing model should be mentioned, i.e. the model developed by Johansen and Just Andersen [8].

Description of packing model

The packing model, which is in focus in this article, is based on a model developed in the light of the principle of packing of binary mixtures and extended to multi-component mixtures, Stovall et al., [9]. The basic model is developed further and modified by incorporating experimentally determined packing, Glavind et al., [10].

The basic packing formula is as follows:

$$\text{Packing} = \underset{i=1}{\overset{n}{\text{Minimum}}} \left(\alpha_i + (1-\alpha_i) \sum_{j=1}^{i-1} g(i,j)\,\phi_i + \sum_{j=i+1}^{n} f(i,j)\,\phi_j \right)$$

where

α	is mono-disperse packing, i.e. packing of equally sized particles
ϕ	is the volume fraction
f(i,j)	is the interaction function for the “wall” effect. Small particles close to a larger particle (or the wall of a container) can not be packed as dense as in the bulk.
g(i,j)	is the interaction function for the effect that appears when the small particles are so large that they cannot fit in between the cavities between the large particles without disturbing the packing of the large particles. This effect is characterised by a so-called μ-value. The μ-value states the maximum size ratio between two particle sizes which allows the small particles to pack in between the large particles without disturbing the packing of the large particles.

The mono-disperse packing is an important parameter in the packing calculation. For spherical particles, the mono-disperse packing equals 0.60-0.64. However, the shape of the aggregates is not spherical. Therefore, the mono-disperse packing is normally less than 0.60-0.64. It is practically impossible to determine the mono-disperse packing experimentally, and the following procedure, which is introduced in Glavind et al, [10], can therefore be followed.

For each material which is investigated in the packing analysis, the packing is determined experimentally, see further ahead. The size distribution of each material is divided into a sufficient amount of fractions, and the mono-disperse packing is determined by iteration so the experimentally determined - and the theoretically calculated - packing agrees.

Note that the mono-disperse packing for a material always will be less than, or equal to, the packing of the material. If a material consists of particles with one size, the mono-disperse packing is equal to the packing. On the contrary, if the size distribution is wide, the difference between the mono-disperse packing and the packing is large.

When mono-disperse packing has been determined for each material, the packing of resulting materials can continue with the packing formula described above. The particle size distribution for each material is divided into a number of fractions. The total volume in each fraction is the sum of volume fractions for all particles. The mono-disperse packing in each fraction is the weighed mean value of the mono-disperse packing of each material.

For a more detailed description of the packing procedure, please refer to Stovall et. al, [9], and Glavind et al., [10].

Computer packing program

The procedure described in the previous section has been translated into a computer program which is commercially available at the Concrete Centre, Danish Technological Institute. The input and the output of the program are shown in table 1. The input and output for each material in the packing analysis and for each packing calculation are included.

Table 1 Input and output to the packing program

	Input	**Output**
Material	Density Grading curve Experimental packing	Mono-disperse packing
Calculation	μ-value Amount of divisions of the grading curve Amount of calculation combinations	Packing diagram for a two-component system Packing diagram for a three-component system Compound grading curve

The density and the grading curve do not need explanation. Measurement of packing is treated in the next section. The μ-value and the mono-disperse packing are described in the previous section. The amount of divisions of the grading curve and the amount of calculation combinations determine the accuracy of the calculation. The user manual to the program, [11], describes in detail how to use the program.

Figure 1 shows a packing diagram with two materials and figure 2 shows a packing diagram with three materials. The result in figure 2 is shown as contour lines in %. The marked point at the figure has a packing density of 84 % and the corresponding material composition is 35 % and 0-4 P, 25 % Gravel 6-16 P and 40 % Gravel 16-32 P. Furthermore, the program can print out compound grading curves for up to 7 materials.

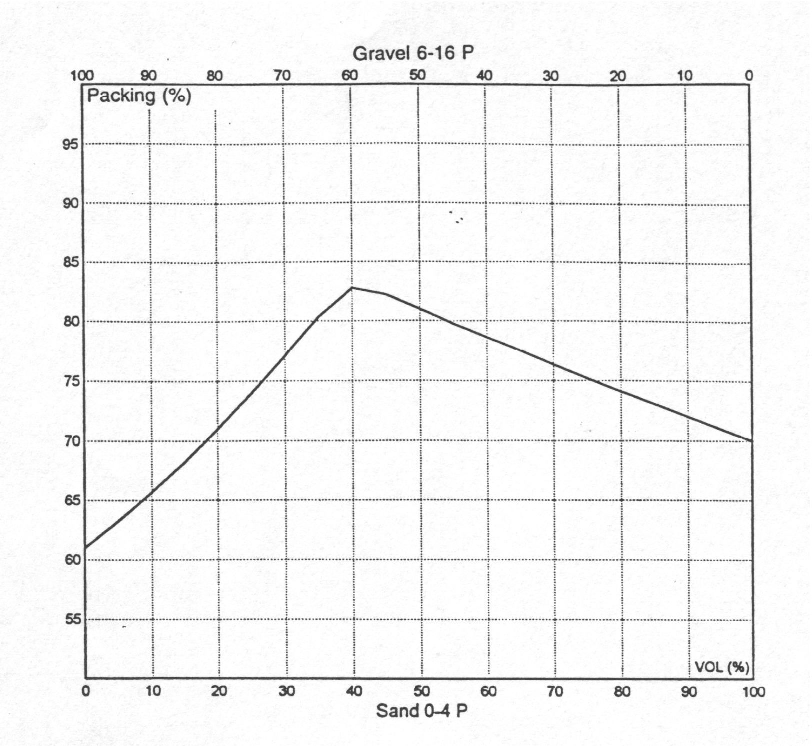

Figure 1 Result of a packing calculation with two different materials (μ=0.7, divisions of the grading curve=20, calculation combinations=8)

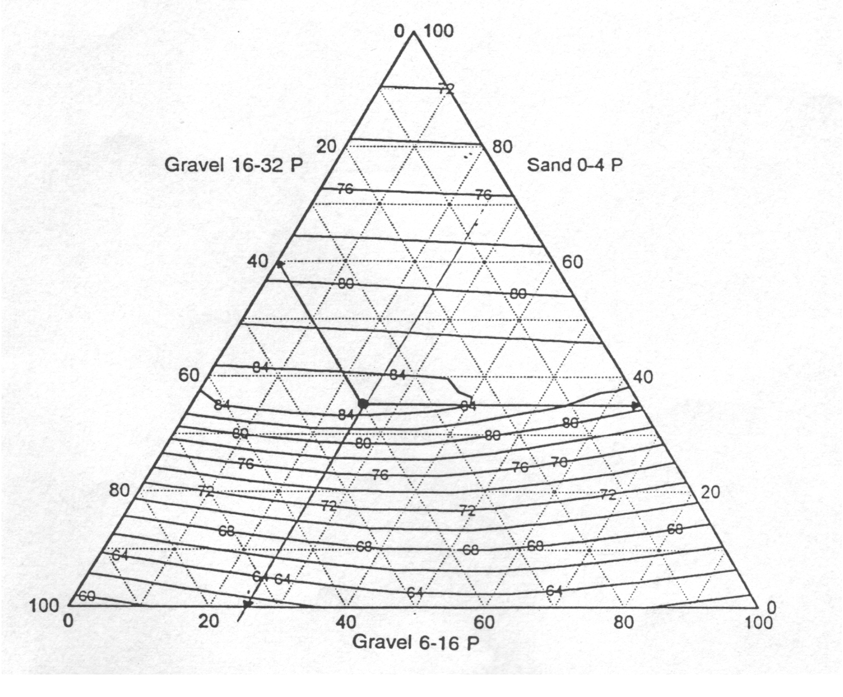

Figure 2 Result of a packing calculation with three materials (μ=0.7, divisions of the grading curve =20, calculation combinations=8).

When a packing analysis is performed for more than three materials, the principle can be illustrated by following the example for four materials. A two-component packing calculation is carried out for two of the four materials. Then a combination of the two materials is selected, and the program is set to treat this combination of the two materials as one material. This principle can be carried out for an infinite number of materials.

Measurement of packing

No standardised method exists which is suitable for determination of the packing of aggregates. Based on experience, it has been found most convenient to pack the aggregates in such a way that the most dense packing is achieved. This is not by vibration, but by a combined shaking-tapping process. The procedure is described in detail in the user manual, [11].

The precision for determination of packing is approximately ± 2 %. This means that for a correct value of 0.60, an interval of 0.59 to 0.61 can be expected.

Table 2 shows typical values for experimentally determined packing and theoretically calculated mono-disperse packing for different Danish aggregate types. It can be seen from the table that a wide particle size distribution results in a high packing value. Furthermore, it can be seen that the round particles (sea and pit materials) obtain higher mono-disperse packing than the sharp-edged particles (granite).

Table 2 Typical values for experimental packing and theoretical mono-disperse packing for different Danish aggregate types.

Material	Particle sizes, mm	Measured packing	Calculated mono-disperse packing
Sea sand	0-1	0.62	0.55
Sea sand	0-4	0.68	0.55
Pit sand	0-4	0.66	0.53
Pit gravel	0-8	0.69	0.53
Sea stone	4-8	0.65	0.59
Sea stone	8-16	0.62	0.57
Sea stone	16-32	0.61	0.56
Pit stone	4-8	0.60	0.55
Pit stone	8-16	0.60	0.55
Pit granite	6-12	0.59	0.51
Crushed granite	8-16	0.58	0.53

CALIBRATION AND VERIFICATION OF THE PACKING MODEL

Sensitivity analysis of μ-value

A μ-value of 0.07 is found to be valid for Danish sea aggregates, while a μ-value of 0.20 is valid for spherical particles. A sensitivity analysis for the μ-value has shown that differences from 0.05-0.10 do not result in large variations in the packing diagram. However, if necessary, it is possible to determine the correct value of μ, [11].

Calculation accuracy

The amount of division of the grading curve and the amount of calculation combinations determine the calculation accuracy and thereby also the time of calculation. Typically, the larger these parameters are, the softer are the contour lines on the packing diagram. The parameters have to be optimised by the user in relation to the desired accuracy and the capacity of the personal computer. However, it must be remembered that it is not possible and it has no meaning - to define an unambiguous optimum from packing calculations. It is areas of optimum packing which can be used for concrete mix design, see later in the paper.

Verification

The model has been verified by packing different combinations of aggregates and compared with packing calculations, Glavind et al., [12]. Even though, only a few experiments have been made, it is found that the packing model describes the packing of aggregates well.

Another kind of verification is packing of aggregates compared with concrete properties. Experience from a number of ready mix factories and concrete element and product factories has shown that the aggregate packing of well functioning concrete is situated in the area (or with a higher sand %) with the maximum packing. This will be described further in the paper.

APPLICATION OF PACKING CALCULATIONS

Packing calculations can be applied as a tool for concrete mix design while starting up new mix designs or materials for production or when optimising existing mix designs:

1. Selection of aggregate types and - amounts
2. Selection of amount of binder
3. Design of air void structure

Selection of aggregate types and -amounts

Experience has shown that the selection of aggregate types and -amount should ensure that the packing is a little above the maximum packing, meaning that the sand % is a little higher than corresponding to the maximum packing. This is in agreement with the findings in Goltermann et al, [13]. Another advantage of this is also that the packing is in the very sensitive area. An aggregate composition in the area below the maximum packing results in very close contour lines, see figure 1, meaning that small variations in the aggregate size distributions and shapes results in large variations in the packing and in the concrete properties.

Another approach is to design the packing so it corresponds with a required binder amount, see next chapter.

The described design approach has been used with success by several Danish concrete producers. As mentioned previously, the aggregate composition for a well functioning mix design almost always is situated in the most optimum packing area. A major investigation at a ready-mix concrete factory has shown that the concrete with the aggregate composition in the optimum area obtain the most optimum fresh concrete properties, and that the concrete with aggregate compositions in the packing area below the optimum obtains bad fresh concrete properties, [12].

Selection of amount of binder

When the aggregate composition has been selected - and thereby a fixed packing -, the amount of binder can be selected so that it corresponds to the cavities between the aggregate. Experience shows, that typically this amount must be increased with 1-4 % more binder. This can be explained with the fact that every aggregate particle must be covered with a layer of paste, and the aggregate particles do not touch each other as assumed in the calculations. Apart from that there must be given room to air voids. A simple rule-of-thumb is that the paste volume can be calculated as

Paste volume = 100 % - packing in % - air void in % + (1-4) %.

As mentioned under the chapter “Selection of aggregate types and amounts”, the opposite situation can be realistic; that the packing and thereby the aggregate type and amount is selected from a required amount of binder, e.g. paste and air voids.

The application approach with designing the amount of binder so it fits with the packing has been used with success by several Danish concrete producers (ready-mix, elements, products) who have analysed a large number of mix designs. It is found that it is typically possible to save some of the cement paste for a constant water-cement ratio and satisfactory workability .

Design of air void structure

A large development project carried out in co-operation with a Danish ready-mix concrete producer has shown that it is possible to design a stable air void system in concrete by optimisation of the composition of the aggregates.

The deficit of paste in relation to cavities between the aggregates (1- packing) determines the total air content. There is a tendency that the aggregate particle size distribution and the air void size distribution relate. The reason for not being able to observe a clear relation is probably that there are large variations on the aggregate particle size distribution.

Furthermore, the physically produced air voids between the aggregate skeleton are more difficult to break down under mechanical influence than chemically produced air voids. Therefore, concrete with physically produced air voids is more stable during pumping and vibration.

The project is more thoroughly described in [13] and in [14]. The project shows very interesting aspects for the concrete producer. However, there can be difficulties with the practical application of the idea. This is because, it is not possible with the existing production facilities to control the particle size distributions precisely enough. In addition such concrete with physically designed air voids can require a change in the execution method, because the concrete is less workable than “normal” concrete.

CONCLUSION

Packing of aggregates can be calculated on the basis of experimental packing, the density and the grading curve for each material.

Packing calculations can be applied as a tool for concrete mix design while starting up new mix designs or introducing new materials for production or when optimising existing mix designs. Practical experience is described for selecting aggregate types and corresponding amount, for selecting the amount of binder, and for designing the air void system.

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